27 $\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx = \lim_{a\to 0^+}\int_a^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx$

$u = x^2+2x$

$du = (2x+2)\,dx$

$du = 2(x+1)\,dx$

$\frac{du}{2(x+1)} = dx$

$\int \frac{x+1}{\sqrt{x^2+2x}}\,dx = \int \frac{\cancel{x+1}}{\sqrt{u}}\,\frac{du}{2\cancel{(x+1)}}$

$\frac{1}{2}\int u^{-1/2}\,du = \cancel{\frac{1}{2}}\,\frac{u^{1/2}}{\cancel{1/2}}$

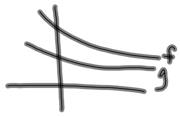
$\lim_{a\to 0^+}\sqrt{x^2+2x}\,\Big|_a^1$

$\lim_{a\to 0^+}\sqrt{3}-\sqrt{a^2+2a} = \sqrt{3}$

Jan 8-8:58 AM

8.4b Improper Integrals

Comparison Test



If $f(x) \geq g(x) \geq 0$

If $\int_a^\infty f(x)\,dx$ converges then $\int_a^\infty g(x)\,dx$ converges

If $\int_a^\infty g(x)\,dx$ diverges then $\int_a^\infty f(x)\,dx$ diverges

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$\int_1^\infty e^{-x^2}\,dx = \lim_{b\to\infty}\int_1^b e^{-x^2}\,dx < \lim_{b\to\infty}\int_1^b e^{-x}\,dx$

$\lim_{b\to\infty}\left(-e^{-x}\right)\Big|_1^b = \lim_{b\to\infty}\left(-e^{-b} - -e^{-1}\right)$

$= 0 + e^{-1} = \frac{1}{e}$

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Does the integral converge or diverge?

$\int_1^\infty \frac{dx}{x^5+1} < \int_1^\infty \frac{dx}{x^5}$ converges

converges by comparison

$\lim_{b\to\infty}\int_1^b x^{-5}\,dx = \lim_{b\to\infty}\frac{x^{-4}}{-4}\Big|_1^b$

$= \lim_{b\to\infty}\left(\frac{1}{-4b^4} - \frac{1}{-4\cdot 1^4}\right) = \frac{1}{4}$

Jan 17-9:00 PM

Find the volume of the solid obtained by revolving the curve about the x-axis $y = xe^{-x}, 0 \le x < \infty$

$$\int_0^\infty \pi r^2\,dx = \int_0^\infty \pi\left(xe^{-x}\right)^2 dx$$

$$\lim_{b\to\infty} \pi \int_0^b x^2 e^{-2x}\,dx$$

$x^2 \quad e^{-2x}$

$2x \quad -\frac{e^{-2x}}{2}$

$2 \quad \frac{e^{-2x}}{4}$

$0 \quad -\frac{e^{-2x}}{8}$

$$\lim_{b\to\infty} \pi \left[-\frac{x^2}{2}e^{-2x} - \frac{2xe^{-2x}}{4} - \frac{2e^{-2x}}{8}\right]_0^b$$

$$\lim_{b\to\infty} \pi \left[\cancel{-\frac{b^2}{2e^{2b}}}^{\,0} - \cancel{\frac{2b}{4e^{2b}}}^{\,0} - \cancel{\frac{2}{8e^{2b}}}^{\,0} + \frac{2}{8e^0}\right]$$

$$\frac{\pi}{4}$$

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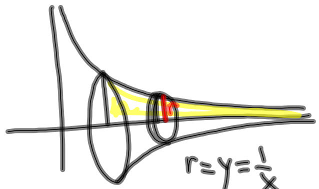
Gabriel's Horn

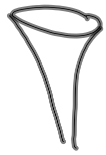
Consider the region R in the first quadrant bounded above by y=1/x and on the left by x=1. The region is revolved around the x-axis.

a) Show that R has infinite area.

b) Find the volume of the solid.

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty} \ln x\Big|_1^b = \infty$$





$$\int_1^\infty \pi r^2\,dx = \int_1^\infty \pi \frac{1}{x^2}\,dx$$

$r = y = \frac{1}{x}$

$$\lim_{b\to\infty} \pi \int_1^b x^{-2}\,dx \qquad \frac{x^{-1}}{-1} = -\frac{1}{x}$$

$$\pi \lim_{b\to\infty} \left(-\frac{1}{x}\right)\Big|_1^b = \pi \lim_{b\to\infty}\left(\cancel{-\frac{1}{b}}^{\,0} - -\frac{1}{1}\right)$$

$$= \pi$$

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